EE3210 Final Exam (Fall 2011). Name (5 pts) ______________________________

5 pages. 100 points. Open book. Open notes. No internet. Cheating will result in a score of 0 for this exam.

Part 1. Short Answer

1. (2 pts) (true/false) If $f(t) \Leftrightarrow F(\omega)$ and $f(t)$ is a discrete time signal, then $F(\omega)$ is periodic.

2. (2 pts) (true/false) If $f(t) \Leftrightarrow F(\omega)$ and $f(t)$ is real, then the magnitude of $F(\omega)$ is even and the phase of $F(\omega)$ is odd.

3. (2 pts) A Butterworth filter is characterized by (circle all that apply)
   (a) equally spaced poles on a circular arc on the left half-plane.
   (b) The sharpest possible transition from the pass band to the stop band.
   (c) ripples in the pass band.
   (d) uniform gain in the pass band.

4. (2 pts) To get the shortest possible rise time, a control system should be
   (a) heavily overdamped
   (b) slightly overdamped
   (c) critically damped
   (d) slightly underdamped
   (e) heavily underdamped

5. (6 pts) Use the properties of the Laplace Transform (and the table) to find the Laplace Transforms of the following signals.

   (a) $f(t) = (t-1)^2 u(t)$

   (b) $f(t) = \sin(2t-2)u(t-1)$

6. (8 pts) A discrete time system is described by $(E^2+0.9E+0.2)y[k]=(E-0.6)f[k]$.

   (a) Find the Transfer function, $H(z)$ ______________________________

   (b) What is/are its pole(s)? ______________________________

   (c) What is/are its zero(s)? ______________________________

   (d) What is/are the characteristic mode(s)? ______________________________

7. (6 pts) State with reason(s) whether or not each of the systems below are asymptotically stable, marginally stable or unstable.

(a) $(E^2 - 1.7E + 0.72)y[k] = \left(E + \frac{1}{2}\right) f[k]$

(b) $(E^2 + 3E + 2)y[k] = (E - 3)f[k]$

(c) $(E^2 - 1)y[k] = (E^2 + E + 1)f[k]$

8. (9 pts) Use the properties of the Z Transform (and the table) to find the Z Transforms of the following signals.

(a) $f[k] = 2^{-k}u[k]$

(b) $f[k] = \cos\left(\frac{\pi}{3}k\right) u[k]$

(c)$f[k] = e^{-2k}u[k - 1]$

Part 2. Problems

9. (5 pts) Find the Inverse Laplace Transform of

$$F(s) = \frac{3s + 10}{s + 7s + 12}$$

10. We wish to design a high pass Butterworth filter with stop- and pass-band edge frequencies f_s=200 and f_p=540. The minimum gain in the pass band is to be $\hat{G}_p$ = -1dB and the maximum gain in the stop band is to be $\hat{G}_s$ = -20dB.

(a) (2 pts) What is the ω_s of the prototype low-pass filter? (Recall that the ω_p of the prototype filter is always 1.)

ω_s = ________________

(b) (3 pts) Find the (minimum) number of poles that this filter requires.

n = __________________________

(c) (3 pts) Find a corner frequency ω_c. State whether your choice of ω_c over-satisfies the gain requirements in the pass band or the stop band.

ω_c = ________________ Over-satisfies _______________________

(d) (3 pts) Find the transfer function of the prototype low-pass filter.

$H(s)$ = _______________________

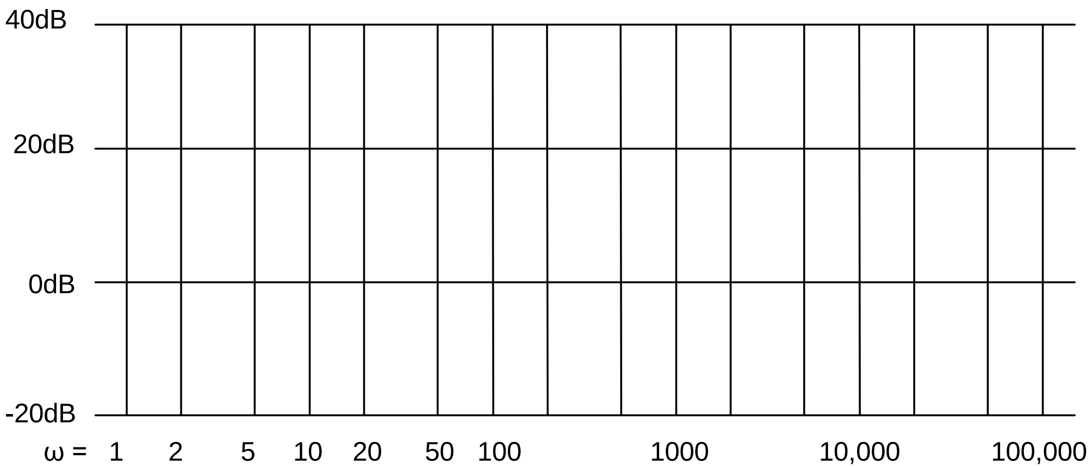
11. (3 pts) Suppose a prototype low-pass filter has a transfer function

$$H(s) = \frac{1.4}{s^2 + 1.5s + 1.4}$$

Find the transfer function of a high-pass filter based on this prototype with ω_p = 100 rad/sec.

12. (5 pts) Draw a Bode Plot (magnitude only) for the frequency response of a system with the given transfer function:

$$H(s) = \frac{25s(s + 200)}{(s + 10)(s + 5000)}$$



13. (5 pts) A discrete time system is described by the equation

$$(E^2 + 0.8E + 0.2)y[k] = (0.1E^2 - 0.3E + 0.5)f[k]$$

Draw a block diagram that realizes this system. Label adder blocks with Σ, label delay blocks with z^{-1}, and label multiplier blocks with the gain. (Hint. Start by writing a difference equation.)

14. (5 pts) Find the inverse Z transform of:

$$F(z) = \frac{2z^2 - 3z}{z^2 - 0.25}$$

15. A discrete time system is described by the equation

$$(E^2 + 2E + 1)y[k] = (E^2 - 4)f[k]$$

(a) (4 pts) Write the difference equation and use it to find the first four data points in the impulse response. (Hint: let $f[k] = \delta[k]$)

h[0] = ________ h[1] = ________ h[2] = ________ h[3] = ________

(b) (4 pts) Find the impulse response, $h[k]$, in closed form, then verify it yields the same data points as in part (a). (Hint: the characteristic modes are $(-1)^k$ and $k(-1)^k$.)

(c) (5 pts) Assume the input $f[k] = (2)^k u[k]$. Find the zero state response, $y[k]$, using the convolution sum. (Yes, it is that ugly, but you don't have to simplify it.)

Problem 15 Continued

(d) (2 pts) Find the transfer function $H(z)$ for this system.

(e) (4 pts) Find the Z-transform, $F(z)$ of the input signal $f[k]$.

(f) (5 pts) Find $Y(z)$ and compute the inverse Z transform to obtain $y[k]$.

Extra Credit (6 pts) A discrete time system is described by the equation

$$(E + 0.25)y[k] = f[k]$$

Find the total response of this system if $y[-1] = 4$ and

$$f[k] = 0.5^k \cos(\frac{\pi k}{3})u[k]$$

Hint. Use z-transform pairs 11a and 11b.